We demonstrate a method based on carrier frequency interferometry (CFI) that measures surface deformation with high accuracy. The method is applied to assess the deformation of thin-film dielectrics deposited on thick substrates. CFI measured the wavefront radius of curvature $R$ with an accuracy of 0.2% for an $R$ smaller than 500 m and 2% for an $R$ between 500 and 2000 m (flat reference substrate). We show the method has a significantly larger dynamic range and sensitivity than Twyman–Green and comparable sensitivity to white light interferometry.

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1. INTRODUCTION

Thin films deposited by physical vapor deposition tend to have large residual stress, which can lead to cracking or peeling of the film. The stress mainly arises from the intrinsic characteristics of the growth process (internal stress) and from differences in the thermal expansion coefficients of the film and substrate (thermal stress). Stress can cause undesirable distortions of the wavefront.

Techniques based on optical interferometry offer the possibility of real-time, remote, non-intrusive, full-field measurements of the curvature and curvature changes in an optic due to stress in the thin-film coatings. In turn, the stress can be calculated from the radius of curvature of the optic ($R$) using Stoney’s equation [1]. However, this is only valid for certain geometrical conditions. Phase-shifting interferometry methods, such as Twyman–Green, work very well when the curvature of the optics is large and the interferogram contains several fringes. This is the case when the substrate thickness is large. When the substrate is thick and the curvature is small, the interferogram may have one or part of one very broad fringe. In such a situation, the fringe analysis returns a value of $R$ with a large error.

Herein, we describe an interferometric method that uses linear carrier frequency interferometry (CFI) to measure the surface deformation and its resulting change in $R$ with high accuracy. We apply the method to measure $R$ samples consisting of a thick substrate in which thin-film coatings are deposited by ion beam sputtering.

This paper is organized as follows. In Section 2, the phase-shifting interferometer, based on Twyman–Green (T-G), is described and examples of the interferograms of thin-film oxides are presented. In Section 3, CFI is described and the results of its application to measure the radius of curvature of the same samples deposited onto substrates of different thicknesses are shown. This section also analyzes the sensitivity of the method. Section 3 describes the application of CFI to measure the radius of curvature of HfO$_2$ thin films onto fused silica substrates of different thicknesses and compares the results with those obtained in the same samples by T-G and white light interferometry. The conclusions section summarize the main features of CFI and discusses its ability to measure stress in thin films.

2. PHASE-SHIFTING INTERFEROMETER

Phase-shifting interferometry (PSI) based on a T-G interferometer [2,3] is schematically shown in Fig. 1.

It uses a broad laser beam, typically an He–Ne laser, as a light source, a micro-objective and a spatial filter, a beam splitter, holders for the sample and the reference plate, a computer-controlled piezoelectric transducer (PZT), and a charge-coupled array detector (CCD).

The beam propagates through a collimating lens to form a planar wavefront. The wavefront is the amplitude divided by the beam splitter. The reflected and transmitted beams travel to a reference sample (fused silica substrate 25.4 mm in diameter, 6.3 mm in thickness) and the test sample. The flatness of the reference sample is within a tenth of the wavelength of the laser beam. After reflection from both the reference and test samples, the beams recombine at the beam splitter and travel toward the CCD. The interference pattern is observed on a
From the measured fringe radius $r$ the radius of curvature recorded image, knowing the camera pixel size) and using

\[ \text{variations in deformation} \]

\[ \text{those in Fig. 2, from a highly curved optic (} \lambda/4 \text{).} \]

\[ \text{The interferograms contain only one fringe (large } R \text{), as shown} \]

\[ \text{in Fig. 3, the uncertainty in } R \text{ can be very large, as mentioned} \]

\[ \text{in the introduction. Thus, PSI T-G is limited in its applications to} \]

\[ \text{highly curved samples.} \]

3. CARRIER FREQUENCY INTERFEROMETRY

To overcome the limitations in sensitivity of the PSI to assess $R$, we applied CFI [3]. CFI was implemented using the geometry shown in Fig. 1 using a beam expander (Thorlabs, model GBE20-A) with a magnification of 20 x and a wavefront error of less than $\lambda/4$. The interferograms were captured using an Andor Neo 5.5 CCD camera, with pixel size 6.5 $\mu$m and 2560 x 2160 active pixels (16.6 mm x 14 mm), which were fully illuminated by the 5-cm-wide expanded He–Ne laser beam. This ensured that only the most intense part of the Gaussian illumination was used. The interferometer’s arms were set equal to 20 cm. The reference plate, 2” in diameter, a fused silica disk, has a flatness $\leq \lambda/4$. The CFI technique requires only a single frame for phase extraction; therefore, it is capable of making high-speed measurements and is insensitive to vibrations. The CFI setup is identical to the PSI T-G interferometer except that in CFI, the reference mirror is intentionally tilted along the $x$- or $y$-axis to create fringes, as shown in Fig. 4.

A tilt about the $y$-axis in an interferogram can be considered to be a linear carrier in the $x$ direction. If the reference wavefront is flat and the wavefront under analysis is also flat, then the fringes are straight, parallel to the $y$-axis, and equidistant. If the wavefront being analyzed is not perfect, then the irradiance function is a nearly a sinusoidal function with phase modulation. The phase modulation is due to the wavefront deformations, $W(x, y)$. If a tilt $\phi$ about the $y$-axis is introduced between the two wavefronts, then the signal (intensity field), $g(x, y)$, can be written as

\[ g(x, y) = a(x, y) + b(x, y) \cos(2\pi f_{0} x + W(x, y)), \]

where $a$ and $b$ are constants.
where $f_0$ is the carrier frequency equal to $f_0 = \sin(\phi)/\lambda$. The coefficient $a(x, y)$ is the background intensity, and $b(x, y)$ is the fringe amplitude.

The wavefront deformation $W(x, y)$ is obtained using a fast Fourier transform (FFT) algorithm [5]. Figure 5 shows the different steps in the interferogram’s analysis. Since the spatial variations of $a(x, y)$, $b(x, y)$, and $W(x, y)$ are slow compared with the spatial frequency $f_0$, the Fourier spectra are separated by the carrier frequency $f_0$. We used one of the two FFT spectra (+ or -) shifted by $f_0$ on the frequency axis [Fig. 5(b)]. Using a filter in the frequency domain and applying the same FFT algorithm, we compute the inverse Fourier transform of the shifted signal. This procedure gives the phase modulation of the deformed wavefront [Fig. 5(c)]. Cross sections of $W(x, y)$ along the x and y radii are shown in Fig. 5(d).

The next step in the analysis requires the application of an unwrapping procedure [6,7]. The result of this procedure yields a three-dimensional image of $W(x, y)$ across an optic, as shown in Fig. 6(a). The cross sections of $W(x, y)$ along the x-axis are shown in Fig. 6(b). The final step in the analysis is to change the wavefront phase values into the sample’s surface deformation $t$ [Eq. (2)]. From the deformation data, $R$ is obtained using Eq. (3).

To test the sensitivity of CFI, fringes were generated and analyzed by the CFI software. The model fringes represent curvatures of 1/30, 1/100, 1/300, 1/500, 1/1000, 1/1500, and 1/2000 m (curvature of the flat reference substrate in the experiments). The results of the analysis are presented in Table 1. The accuracy of the analysis is of order of 0.5% for smaller $R$ ($R < 100$ m) and increases to 2% for larger $R$ ($R > 500$).

Table 1. Analysis of the CFI Sensitivity in the Determination of $R$

<table>
<thead>
<tr>
<th>Interferograms for $R$ [m]</th>
<th>CFI Analysis $R$ [m] ($R_x/R_y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>29.9/30</td>
</tr>
<tr>
<td>100</td>
<td>99.6/99.8</td>
</tr>
<tr>
<td>300</td>
<td>295.5/301.4</td>
</tr>
<tr>
<td>500</td>
<td>491.6/502.7</td>
</tr>
<tr>
<td>1000</td>
<td>992/995.5</td>
</tr>
<tr>
<td>1500</td>
<td>1498/1501.5</td>
</tr>
<tr>
<td>2000</td>
<td>1987/2011</td>
</tr>
</tbody>
</table>

There are three main sources of errors that can influence the determination of $W(x, y)$:

(a) The quality of the optical elements used in the interferometer. The accuracy of the system is limited by the most inaccurate optical element. In the system presented in this paper, the wavefront errors (WFEs) of the beam expander, beam splitter, and reference optics are $\leq \lambda/4$ at 633 nm.

(b) Optical noise (dust, scattering, stray reflections, poor fringe contrast, speckle noise) and detector nonlinearity.

(c) The density of the carrier fringes.

The density of the carrier fringes affects the accuracy of the interferometer in the following ways. The interference fringes introduced by the tilt of the reference mirror (carrier frequency $f_0$) spatially sample the phase function (searched $W(x, y)$). In general, $f_0$ must be equal to or larger than the frequency of $W(x, y)$. For a very flat wavefront (large $R$), any minimal tilt (any minimal $f_0$) that separates the zero and first orders in the Fourier domain will provide an accurate determination of $R$. For very curved samples (small $R$), it is very easy to reach the Nyquist limit [8] and that will limit the measurement. A large number of fringes may not be registered because of the limited spatial resolution (pixel size) of the detector. In our system, the Andor CCD camera has a pixel size of 6.5 micrometers. Taking into consideration the Nyquist limit (2 pix/fringe, but in practice, it is 4 pix/fringe), the measurement cannot be performed when there are more than 630 fringes on the interferogram. In practice, the number of recorded fringes may be lower because of optical noise and detector nonlinearity. The minimum $R$ that can be measured with our system is around 5 m.

The other consideration in the fringe analysis arises from the limitations imposed by the aperture defined in the Fourier domain to separate the ±1 orders with respect to the zeroth order. As shown in Fig. 7 for $R = 10$ m and $R = 1$ m, the Fourier signals are well separated from the center zeroth order and the method can be applied. Instead, for the example of $R = 0.5$ m, the high density of fringes in the interferogram generates an image in the Fourier plane, in which the spreading of the signal in the frequency domain leads to broadened ±1 orders. An extreme situation is shown for $R = 0.3$ m, where there is significant overlap in the Fourier spectrum of the ±1 orders. In these cases, CFI cannot be applied. Figure 8 presents an interferogram of a sample with $R = 10$ m, recorded in our system. The analysis retrieves a value of $R = 10.8$ m. One can see that the first-order spectrum is broadened because of noise.
CFI was used to obtain full-field maps of the curvature of fused silica substrates 25.4 mm diameter ($2r$) and coated with a 160-nm-thick layer of HfO$_2$ grown by dual ion beam sputtering. Fused silica substrates of different thicknesses were used to highlight the higher sensitivity of CFI compared to T-G. The CFI results were also compared with measurements performed with a ZYGO -The NewView 7300 white light interferometer (WLI). The results, summarized in Table 2, show CFI accurately determines the curvature of the optics when compared to WLI-ZYGO. The results also show the advantages of CFI to determine $R$ when $r/t_s$ is small (i.e., thick substrates), and the T-G method returns values of $R$ with large errors or is incapable of measuring it.

5. CONCLUSIONS

We have implemented CFI using a simple optical system and showed that the method can measure the wavefront radius of curvature of thin-film optics deposited on thick substrates. The wavefront is determined from a Fourier analysis of the fringe pattern image. We showed CFI can determine $R$ in the range of $10$–$2000$ m (flat substrate) with a 0.5% accuracy for $R < 100$ m and 2% accuracy for $R > 500$ m. The values of $R$ agree within 10% with respect to the values determined by WLI-ZYGO. This is in contrast to T-G, which produces values of $R$ with large errors, even when the substrate thickness is 1 mm, and is incapable of retrieving $R$ for the same sample deposited in thicker substrates. The advantage of CFI compared to T-G is the ability to control the density of fringes in the interferogram to suitably map the slower spatial variations of the wavefront, thus significantly increasing the range of values of $R$ that can be measured. This capability makes CFI a valuable method to measure wavefront variations in optical elements of laser systems.

To ensure maximum accuracy in the determination of the radius of curvature, we have developed a measurement protocol whereby each substrate is first measured in order to determine its curvature. Next, the measurement is repeated after the substrate is coated and experiences changes in curvature. The difference in both values gives the absolute sample's deformation. An analysis is in progress to relate wavefront modifications to stress. Stress is obtained from the wavefront radius using the Stoney equation, which provides values of the film stress with 5% error as long as the total thickness $t_s + t_f$ is as least 11 times smaller than the shortest side of a rectangular plate and at least 9 times in case of a circular plate ($R/(t_s + t_f) > 9$) [9,10]. This criterion is not valid for the $6.3$ mm thick, $12.7$ mm radius substrates used in this work (ratio $R/(t_s + t_f) = 2.05$). Therefore, the error in the stress value from the Stoney analysis can be more than 30%. It is important to point out that when meeting the geometrical requirements of the Stoney, CFI could more accurately determine stress in coatings compared to T-G due to its higher sensitivity and dynamic range.

**Table 2. Comparison between Results Obtained in Carrier Frequency Interferometry (CFI), White Light Interferometry (WLI-ZYGO), and Twyman–Green (T-G) Interferometry**

<table>
<thead>
<tr>
<th>Sample</th>
<th>$t_s$ [mm]</th>
<th>$r/t_s$</th>
<th>$R$ [m]</th>
<th>CFI</th>
<th>WLI-ZYGO</th>
<th>T-G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substrate 1</td>
<td>6.3</td>
<td>2.01</td>
<td>2330</td>
<td>2570/2672</td>
<td>No results</td>
<td></td>
</tr>
<tr>
<td>Substrate 2</td>
<td>6.3</td>
<td>2.01</td>
<td>1600</td>
<td>1883</td>
<td>No results</td>
<td></td>
</tr>
<tr>
<td>Sample A</td>
<td>6.3</td>
<td>2.01</td>
<td>2100</td>
<td>2237/2157</td>
<td>No results</td>
<td></td>
</tr>
<tr>
<td>Sample B</td>
<td>2.03</td>
<td>6.26</td>
<td>2170</td>
<td>2115</td>
<td>No results</td>
<td></td>
</tr>
<tr>
<td>Sample C</td>
<td>1.05</td>
<td>12.1</td>
<td>315</td>
<td>324</td>
<td>520</td>
<td></td>
</tr>
<tr>
<td>Sample D</td>
<td>0.51</td>
<td>25.0</td>
<td>65</td>
<td>62</td>
<td>72</td>
<td></td>
</tr>
</tbody>
</table>

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